Q1. Write out a formal proof by mathematical induction that the maximum number of regions the plane is divided into by n lines is 1/2 (n2 + n + 2).

Q2. By the mathematical induction, prove that

32n+1+(-1)n2=0(mod 5)

Q3. Prove using mathematical induction that for all n ≥ 1,

1 + 4 + 7 + · · · + (3n − 2) = n(3n − 1)/2

Q4. Use the Principle of Mathematical Induction to verify that, for n any positive integer, 6n – 1 is divisible by 6.

Q5. Verify that for all n ≥ 1, the sum of the squares of the first 2n positive integers is given by the formula

12 + 22 + 32 + · · · + (2n)2 = (n(2n + 1)(4n + 1))/3

Q6. Use the Principle of Mathematical Induction to verify that, for n any positive integer 22n – 1 is divisible by 3.

Q7. 2n + 1 < 2n, for all natual numbers n ≥ 3

**Scenario Based**

Q1. Assume that an isolated island has a fixed population. Out of these, k number of people have red eyes, and the rest of the people have green eyes. No one on the island knows their eye color but knows every other person on the island. If a person on the island ever discovers they have blue eyes, that person would be outcast and leave on the first night. If it cannot be determined, they can stay on the island. There are no reflective surfaces, and there is no communication of eye color.

One day, some foreigners anchor their ship on the island. They call all the islanders and announce that at least one of them has red eyes. Naturally, the announcement is assumed to be true since it is common knowledge that foreigners are truthful. Following this logic, it is thus common knowledge that at least one islander has red eyes.

#### Ans. Solution

The straightforward answer to the above problem would be that all the red-eyed people will be outcast on the kth night after the announcement.

As for how? By following the principle of mathematical induction, if k = 1, there is precisely one red-eyed islander; the person will realize that he alone has red eyes since he already knows the eye color of every other islander. If k = 2, then with the principle of common knowledge, no one will leave on the first night. The two red-eyed people, seeing only one person with red eyes and that no one left on the 1st night (and thus that k > 1), will go on the 2nd night. Through the principle of induction, it can be reasoned that no one will leave at the first k − 1 night if and only if there are at least k red-eyed people. Those with red eyes, seeing k − 1 red-eyed people among the others and knowing there must be at least k, will conclude that they must have red eyes and leave.

In this particular scenario, the common knowledge principle comes into effect. The common knowledge principle is nothing more or less than what the islanders already know. Meaning, when the foreigners announce that at least one of them has red eyes, it becomes common knowledge since there was no way for the islanders to communicate the same. Hence, before this fact is announced, the truth is not comprehensible by the islanders; hence is not common knowledge. But once it is conveyed to them by the foreigners, it becomes common knowledge. Thus a fact that becomes of common understanding has an evident and noticeable effect. When the foreigners publicly announce a presence already known to all but not evident becomes common knowledge, the red-eyed people on the island eventually deduce their status and leave.

As we have seen, mathematical induction explains real-life scenarios logically. It is an essential part of algebra and other parts of mathematics and helps solve critical questions that we experience in daily life through test, assumption, proof, and explanation.

Read about the recent developments in mathematical induction. Find out more on related topics such as Applications of PMI in Proving Divisibility Rules and relation of Fibonacci sequence with installation, and many more.